

Reconstructing Dark Energy

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The discovery of the accelerated expansion of the universe poses perhaps the greatest puzzle in fundamental physics today. A solution of this problem will profoundly impact cosmology and could also provide key insights in reconciling gravity with quantum theory. Driven by these motivations, the fundamental aim of ground and space-based missions such as the Baryon Oscillation Spectroscopic Survey (BOSS), the Dark Energy Survey (DES), and the Large Synoptic Survey Telescope (LSST), to name just a few, is to unravel the secret of cosmic acceleration. In search of the underlying explanation, theoretical approaches fall into two main categories: (1) dark energy—invoking a new cosmic ingredient, the simplest being a cosmological constant, and (2) modified gravity—invoking new dynamics of space-time.

Even if we choose not to modify general relativity and restrict attention to dark energy, a fundamental difficulty is the absence of any single compelling theory to test against observations. Thus, to avoid dealing with a zoo of models, a phenomenological approach is typically followed where one studies the behavior of dark energy by parameterizing its equation of state parameter, w , the ratio of the pressure to the density. For different dark energy models, w is different, and dynamical origins of dark energy, such as quintessence fields, lead to a time variation in the equation of state. Analysis efforts therefore focus on characterizing this time dependence. Current observations are consistent with a cosmological constant, that is, with having $w = -1$, at the 10% level. The implied value of the cosmological constant is, however, in utter disagreement with theoretical estimates of the vacuum energy, being too small by a factor of at least sixty orders of magnitude. It is therefore an ad hoc addition with no hint of a possible origin, hence the focus on dynamical explanations, such as field theory models or modified gravity. Although detection of any time or, equivalently, redshift dependence in $w(z)$ would immediately rule out a cosmological constant, such observational imprints must necessarily be subtle, otherwise they would have been discovered already. This is the central motivation behind our work: the construction of a robust framework with controlled error bounds that can reliably extract $w(z)$ from diverse observational datasets [1,2].

As a first example, we focused on the supernova Ia light-curve data used to determine the cosmological luminosity distance-redshift relation. For this dataset, the reconstruction task is equivalent to inverting the action of a nonlinear smoothing operator involving a double integral, a classic statistical inverse problem. The two commonly used approaches are

either to parameterize $w(z)$ with simple functional forms, or to employ principal component analysis (PCA), to work with eigenmodes defined as linear combinations of bins. The problem with the first method is a susceptibility to bias if the data is not well-represented by the assumed functional form, while the second method can force an unphysical view of $w(z)$ because the actual $w(z)$ is not piecewise constant. In contrast, our nonparametric approach employs a distribution over random functions to represent $w(z)$, and estimates the statistical properties thereof, given observed data, using Markov chain Monte Carlo (MCMC).

We use Gaussian Process (GP) modeling to represent random function realizations. GPs extend the multivariate Gaussian distribution to function spaces, with inference taking place in the space of functions. The defining property of a GP is that the vector that corresponds to the process at any finite collection of points follows a multivariate Gaussian distribution. Gaussian processes are elements of an infinite dimensional space, and can be used as the basis for a nonparametric reconstruction method. GPs are characterized by mean and covariance functions, defined by a small number of hyperparameters. The covariance function controls aspects such as roughness of the candidate functions and the length scales on which they can change; aside from this, their shapes are arbitrary. Bayesian estimation simultaneously evaluates the GP hyperparameters (so-called to prevent confusion with the parameters that define a parametric method) together with quantities of physical interest. Importantly, in our specific application, the use of GPs allows us to take advantage of the particular integral structure of the smoothing operator [1].

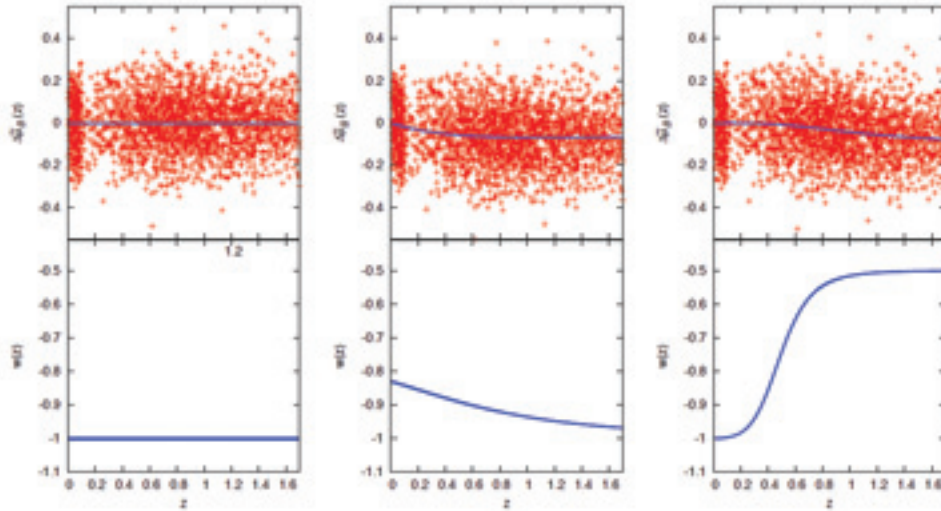


Fig. 1. Three simulated datasets: the upper panel shows the simulated supernova data with the corresponding value for a model with cosmological constant subtracted, the blue line shows the underlying exact (no error) curve. The lower panel shows the behavior of the corresponding $w(z)$ with redshift. The model in the leftmost panels is a cosmological constant, while the next two panels represent two nontrivial quintessence models.

To show how difficult the reconstruction problem is, three simulated datasets—with high quality data—are shown in Fig. 1. Even though $w(z)$ in all three cases is quite different, the associated datasets for the supernova distance modulus are complex and not easy to distinguish, as their differences are quite marginal, and masked by the intrinsic variance of the data. Nevertheless, our technique succeeds in capturing the true behavior of $w(z)$ [1], as shown in Fig. 2.

After testing our procedure on trial datasets, we applied it to current supernova data from a recent compilation [3]. The results are shown in Fig. 3 [2]. Although the errors increase with redshift, the results are in good agreement with a cosmological constant, $w = -1$, and do not show any evidence for a systematic trend with redshift. Nonetheless, the current error limits are not very narrow and do still allow for substantial variation.

We are extending our methodology to systematically include other observations such as the cosmic microwave background temperature anisotropy and baryon acoustic oscillations. As the quality, quantity, and variety of cosmological datasets improves, our methodology will provide ever-tighter constraints on the nature of dark energy.

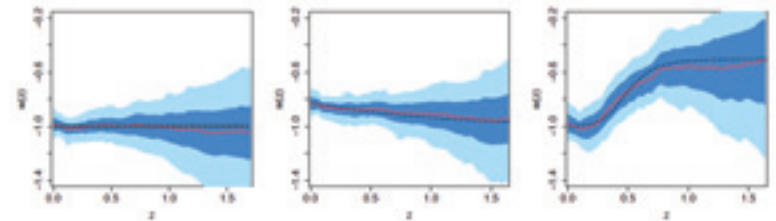


Fig. 2. The $w(z)$ behavior as reconstructed by our nonparametric GP model-based approach [1] from the simulated datasets shown in Fig. 1. Error bars increase at higher z due to sparser supernova sampling.

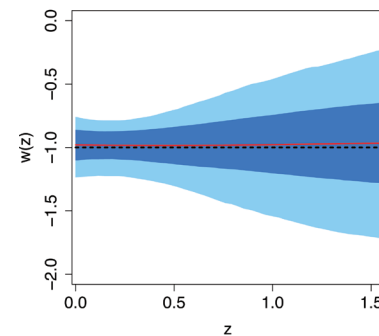


Fig. 3. Reconstruction of $w(z)$ [2] from a recent compilation of supernova data [3]. Note that there is no evidence for any systematic trending away from a cosmological constant, although the allowed variability is not small.

[1] Holsclaw, T., et al., *Phys Rev D* **82**, 103502 (2010).

[2] Holsclaw, T., et al., *Phys Rev Lett* **105**, 241302 (2010).

[3] Hicken, M., et al., *Astrophys J* **700**, 1097 (2009).

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